



Analysis of cycloid features by video analysis

*Análise das características da Ciclóide por vídeo-análise*Regiley Pimenta Ramos de Oliveira  and Samuel Bueno Soltau  *

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We analyze and discuss the agreement between the experimental data obtained with video analysis and the theoretical calculations for the classical problem of brachistochrone and tautochrone cycloids. We use the open source software Tracker to examine video segments with simultaneous records of both straight and cycloid ramp to taken experimental data and compare them to the theoretical study. The results are in accordance with the theoretical description. The study of this classical problem is a proof of concept of the reliability and accuracy of the method that can be applied in other classical mechanics problems.

Keywords: Cycloid; Brachistochrone; Tautochrone; Video analysis.

Analisamos e discutimos a concordância entre dados de experimentos obtidos com vídeo análise e os cálculos teóricos efetuados para o problema clássico da braquistócrona e da tautócrona. Usamos o software livre Tracker para examinar segmentos de vídeo contendo registros simultâneos de rampa reta e de cicloide para obter dados experimentais e compará-los com o estudo teórico. Os resultados obtidos concordaram com a descrição teórica. O estudo deste problema clássico serve como prova de conceito da confiabilidade e precisão do método para aplicação em outros problemas de mecânica clássica.

Palavras-chaves: Cicloide; Braquistócrona; Tautócrona; Vídeo análise.

I. INTRODUCTION

Which is the fastest path for a free particle, under only influence of the gravitational field, can travel between two points was a problem proposed, in 1696, by Johann Bernoulli (1667-1748) as a challenge to the mathematicians of his time. Responded to Bernoulli's challenge, Isaac Newton (1643-1727), Gottfried Wilhelm Leibniz (1646-1716), and the Bernoulli brothers [1]. The solution to this problem introduced another problem itself, tautochrone problem: two particles released at different heights in the cycloid reach the end of the cycloid at the same time [2]. It is describe in more detail in the Section II.

The cycloid curve can be described as a circumference segment [3]. The brachistochrone was very studied over the years, and different ways to solve wore found, such as in [4]

where Fermi's Principle is used. In this paper we are use the variational solution of the brachistochrone problem, described at [5]. We use video analysis, in the same way that was performed at [6], but to analyze the properties of the cycloid curve. We use a simpler equipment and then confront the results with the theoretical calculations.

The cycloid problem can also be related with proper time in Schwarzschild's solution of General Relativity equations, when describing a free particle falling straight in toward center gravitational attraction of negligible dimensions. The cycloid relation emerges from the time to fall, hold in Newton's non relativistic theory of gravitation, except that in this case it is ordinary time related and no to proper time [7].

Video analysis consists of taking a video of a phenomenon or experiment and then performing a detailed analysis on it using tools that relate the phenomenon to be studied with (ob-

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servable) physics quantities and its quantification. Through the analysis of the video it is possible to study physics quantities such as position, velocity, acceleration and energy of a body [8, 9].

The videos are a group of pictures (frames) that are shown sequentially, giving the illusion of motion. Generally the videos are recorded at a rate of 30 frames per second, or approximately 0.0333 seconds per frame. In this work, we use the GNU software Tracker [10] that take video frames, that can be easily analyzed and tracked by the researcher.

Some requirements must be observed in order to obtain good videos to conduct successful video analysis, among them, we remark: (i) make sure that the event scene gets perpendicular in relation to a camera lens; (ii) must be a good contrast between the target object and the scene background; and, (iii) the scene be luminosity enough to tracking the target object in every movie frame.

This paper is organized as follows: In the Section II we discuss the math and physics of the cycloids. The proof that the cycloid is a brachistochrone and a tautochrone; Section III are presented the equipments and how the data was tacked; Section IV we show the results and discussions, and to finish, the last section presents the conclusions.

II. THE MATH AND PHYSICS OF CYCLOIDS

II.1. Brachistochrone

To study the brachistochrone problem, it is important to describe the motion. Lets consider the direction to the acceleration of gravity, g , positive and the energy need to be conservative, thus we got:

$$mgy = \frac{mv^2}{2} . \tag{1}$$

here m is the mass of object, y is the ordinate axis, v is the velocity.

We are interested in time, and brachistochrone have shortest time, so it is important

to see where it came from:

$$dt = \frac{dS}{v} , \tag{2}$$

where dS is the infinitesimal part of the circuit.

Substituting v in Eq.(1) by Eq.(2) and integrating in both sides, the equation turns into a line integral, where dS correspond now $(dx^2 + dy^2)^{1/2}$ giving us:

$$T = \int_a^b \sqrt{\frac{dx^2 + dy^2}{2gy}} . \tag{3}$$

We are going to use an mathematical artifice by multiplying it to dy/dy [11]:

$$T = \frac{1}{\sqrt{2g}} \int_a^b \sqrt{\frac{x'^2 + 1}{y}} , \tag{4}$$

where $x' = dx/dy$.

The problem is now a problem of variational principle. Using the Euler-Lagrange equation [12], assuming that $f(x, x', y) = [(x'^2 + 1)/y]^{1/2}$, we finally solve the problem of brachistochrone, and a parametric curve of a cycloid is given:

$$y = a(1 - \cos \theta), \tag{5}$$

$$x = a(\theta - \sin \theta), \tag{6}$$

and it gives the curve shown in Figure 1:

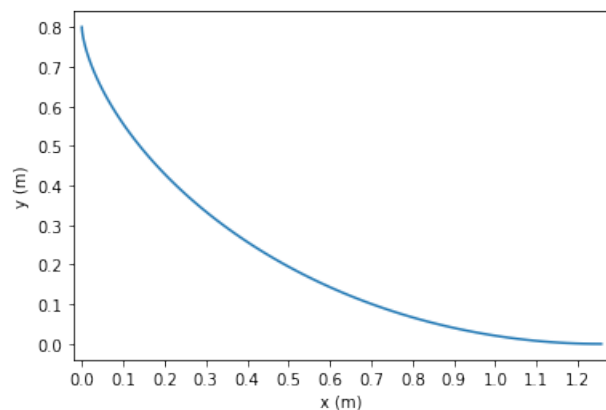


FIGURE 1. Cycloid, where a in equations (5) and (6) are equal 0.8 and the y is inverted and translated to be positive.

II.2. Tautochrone

With this result, we can show the property of the tautochrone, namely, that no matter the position where we put the ball in the cycloid, it will reach the lowest part in the same time interval. Using the Eq.(3), and substituting the dx and dy for the parametric equations (5) and (6), we get:

$$T = \frac{1}{\sqrt{2g}} \int_0^\pi \sqrt{\frac{a(1 - \cos \theta)^2 + a^2 \sin^2 \theta}{a(1 - \cos \theta)}} d\theta. \tag{7}$$

So by solving the integral, we can see that the time is a constant number [13]:

$$T = \sqrt{\frac{a}{g}} \pi. \tag{8}$$

To show that the cycloid is a tautochrone, we can modify the height of the start in y to $y - y'$ in Eq.(3):

$$T = \sqrt{\frac{1}{2g}} \int_a^b \sqrt{\frac{dx^2 + dy^2}{y - y'}} , \tag{9}$$

and replace the variables to equations (5) and (6):

$$T = \sqrt{\frac{1}{2g}} \int_a^b \sqrt{\frac{a(1 - \cos \theta)}{\cos \theta - \cos \theta'}} d\theta, \tag{10}$$

and solving the integral, we get the same value of Eq.(8).

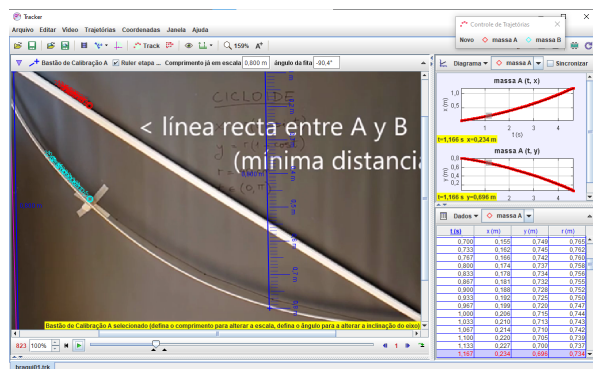


FIGURE 2. A print of Tracker with the points of those balls tracked in a brachistochrone experiment.

III. METHODS AND MATERIALS

To perform the video analysis, the software Tracker [14] and a video with the characteristics described in Section I were used.

For this study, we used a video available in the YouTube platform [15] that shows the events. We analyzed it using the Tracker, when the two balls were tracked along different paths. Figures 2 and 3 show the video analyzer.

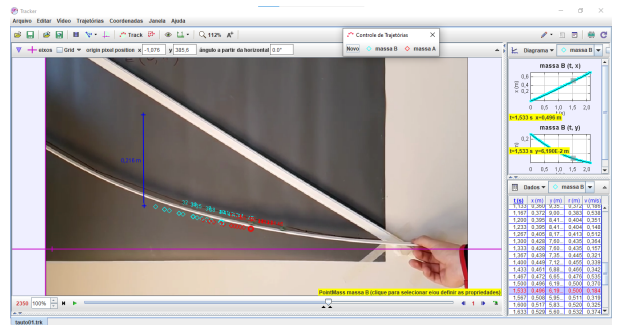


FIGURE 3. A print of Tracker with the points of those balls tracked in a tautochrone experiment.

In the Figure 3 we show the video analysis to take measurements to tautochrone experiment.

The video was taken following the 30 frames per second requirement and so Tracker was able to automatically convert the pixels distance of a frame to the metric system.

Data were collected using the automatic tracker and manual tracker software capabilities as the balls were sometimes merged with the background and the automatic tracker tool does not work under these conditions, requiring human intervention.

IV. RESULTS AND DISCUSSIONS

The data returned by the Tracker software was used to fit the parametric equations (5) and (6) and plotted in Figure 4.

In Figure 4 it is possible to see that the comparison between the cycloid curve obtained by video analysis and the theoretical curve shows good agreement.

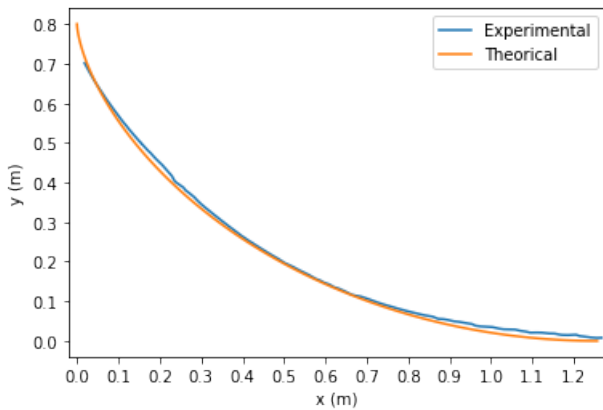


FIGURE 4. Data for video analysis and theoretical curve. The theoretical curve is the same as that Figure 1 with the same parameters.

IV.1. Cycloid vs. inclined plane

The first experiment performed was to analyze the brachistochrone problem by comparing the cycloid and the inclined plane. Both balls started at inertial rest and were released at the same time. The collected data were taken along the paths. The result of the collect comparing the inclined plane and the cycloid curve was plotted in Figure 5.

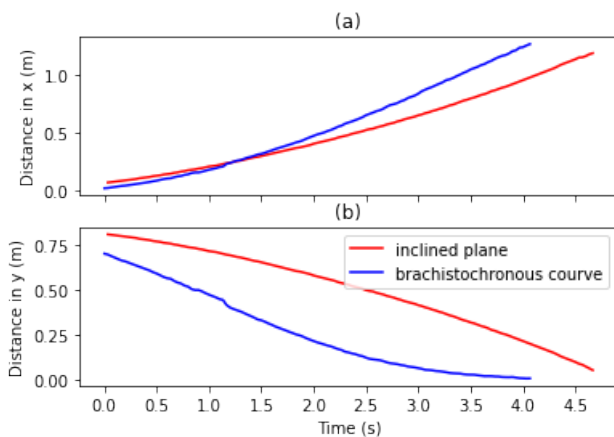


FIGURE 5. (a) Data for brachistochrone curve and inclined plane in x direction. (b) is the data for brachistochrone curve and inclined plane in y direction.

Examining Figure 5 it is evident that the descent of the ball through the inclined plane

takes longer to reach the end point when compared to the cycloid curve that corresponds to the theoretical solution shown in Subsection II.1 more explicit, to see the difference between the cycloid curve and the inclined plane, is shown in Figure 6.

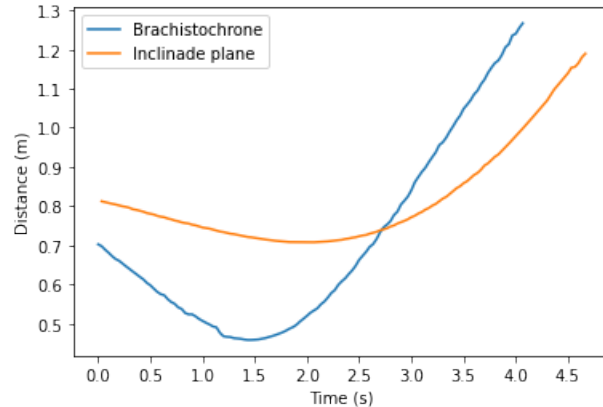


FIGURE 6. Time versus relative distance for two trajectories, cycloid and inclined plane.

Due to the video quality, some points on the curve do not match with the line. During the tracking process, a blur was seen instead of the ball, and manually track was necessary. In general, it is easy to see the ball in the cycloid goes faster than the ball in the inclined plane.

IV.2. Cycloid and two balls

In this part of the study, the tautochrone event is at the focal interest. The event can be represented by Figure 3, where a frame of the video is shown. The two balls start running at the same path and time, but in different heights. Both of them arrive at lowest point in the same time.

In the Figure 7 we can see that both curves meet at the final point of the trajectory. This part of the video was recorded in slow motion, and some frames were doubled causing the points to fall outside expectations. However, this small disturbance in data acquisition did not prevent satisfactory results to support the analysis.

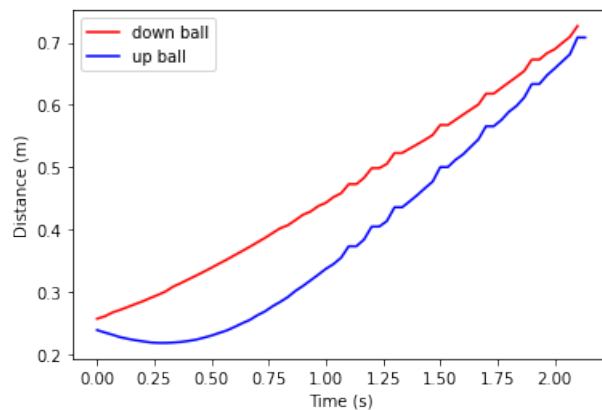


FIGURE 7. Time versus distance in global direction.

V. CONCLUSIONS

Revisiting classical problems with modern methods provides both an effective way to test the accuracy of new methods and to refocus and re-examine old problems in a new sight.

In this paper we explore the experiment comparing the motion of balls in both straight and curved inclined planes to highlight the properties of brachistochrone and tautochrone

cycloid trajectories.

We used the experimental technique of video analysis, which is a simple and low cost way of analysis. We used digital videos acquired from the web and performed under careful procedures that ensure the quality and accuracy of the measurements.

The study proceeded to a two-dimensional analysis of the downward motion of balls of equal mass along two-dimensional ramps. Such a method has advantages when compared with other instruments and bench-top experimental setups, such as those using stopwatches, photocells, in conventional setups.

In summary, the video analysis of this classical problem using a free software presents itself as a proof of the concept and of the potential uses that the method has to be applied to other classical problems and in generic mechanic experiments.

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- [1] L.C. Machado, *A Braquistócrona*. Monografia (Especialização), Instituto de Ciências Exatas da UFMG, Belo Horizonte, (2011).
 - [2] H.W. Broer, *Bernoulli's light ray solution of the brachistochrone problem through Hamilton's eyes*. Int. J. Bifurc. Chaos. **24**, (8) 1440009 (2014).
 - [3] C.H. de S. Marculino, V.J. Pimenta, F. de F. dos S. David, et al., *Uso de vídeo-análise para promover a experimentação dos conceitos de Física com atividades ao ar livre*. Rev. Ens. Ciências e Matemática **10**, (3) 243 (2019).
 - [4] H. Erlichson, *Johann Bernoulli's brachistochrone solution using Fermat's principle of least time*. Eur. J. Phys. **20**, (5) 299 (1999).
 - [5] J.R.A. de Sousa Júnior, *O cálculo variacional e o problema da braquistócrona*. Dissertação - Instituto de Geociências e Ciências Exatas da UNESP, Rio Claro (2010).
 - [6] P. Onorato, P. Mascheretti, A. Deambrosis, *Studying motion along cycloidal paths by means of digital video analysis*. Eur. J. Phys. **34**, (4) 921 (2013).
 - [7] C.W. Misner, K.S. Thorne, J.A. Wheeler, *Gravitation*. New Jersey: Princeton University Press (2017).
 - [8] P. Laws, H. Pfister, *Using digital video analysis in introductory mechanics projects*. Phys. Teach. **36**, (5) 282 (1998).
 - [9] R. Beichner, *The impact of video motion analysis on kinematic graph interpretation skills*. Am. J. Phys. **64**, 1272 (1996).
 - [10] D. Brown, *Tracker Video Analysis and Modeling Tool*, Version 4.751. (2013). <https://www.compadre.org/osp/items/detail.cfm?ID=7365> (accessed on August 12, 2021).
 - [11] D. Figueroa, G. Gutierrez, C. Fehr, *Demonstrating the brachistochrone and tautochrone*. Phys. Teach. **35**, (8) 494 (1997).
 - [12] P.C. Deshmukh, P. Rajauria, A. Rajans, B.R. Vyshakh, S. Dutta, *The brachistochrone*. Resonance **22**, (9) 847 (2017).
 - [13] M. Desaix, D. Anderson, M. Lisak, *The brachistochrone problem: an introduction to*

- variational calculus for undergraduate students*. Eur. J. Phys. **26**, (5) 857 (2005).
- [14] <https://physlets.org/tracker/>. (accessed on August 12, 2021).
- [15] https://www.youtube.com/watch?v=_J_MZUF0HsY. (accessed on August 12, 2021).